Presentation of XLiFE++

Calculation of singular integrals

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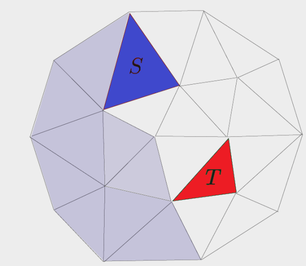
Discretization



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$$\int_{S \times T} w_i(x) \frac{\partial G(x,y)}{\partial n_y} w_j(y) d\gamma_x d\gamma_y$$

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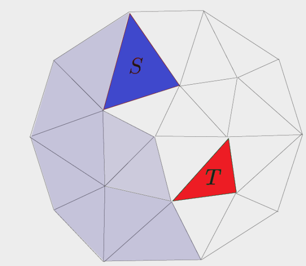

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Green kernels (Laplace, Helmholtz)

$$G(x,y) = \begin{cases} \frac{1}{2\pi}\ln(\ \|x-y\|\ ) \text{ in } 2-D, \\ \dfrac{1}{4\pi\ \|x-y\|} \text{ in } 3-D \end{cases}$$

$$G(x,y) = \begin{cases} \frac{1}{4i}H_0^{(1)}(k\ \|x-y\|\ ) \text{ in } 2-D, \\ \frac{1}{4\pi}\dfrac{e^{ik\ \|x-y\|}}{\|x-y\|} \text{ in } 3-D \end{cases}$$

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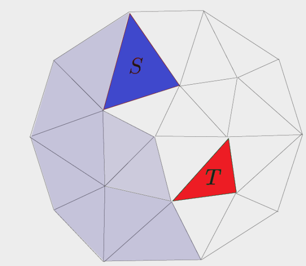

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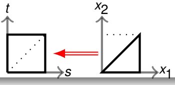
Duffy's transformation

- Change of variables in order to regularise the integrand thanks to the Jacobian:

$$\int_0^1 \int_0^{x_1} \frac{1}{\sqrt{x_1^2 + x_2^2}} dx_1 dx_2 = \int_{[0,1]^2} \frac{s}{\sqrt{s^2 + s^2 t^2}} dsdt = \int_{[0,1]^2} \frac{dsdt}{\sqrt{1 + t^2}}$$

with $x_1 = s$ and $x_2 = st$, so $|J| = s$



Jean Gay's method (CEA).

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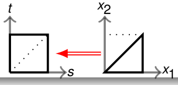

Sauter and Schwab

- Use of the reference element.
- Change of variables ($z = y - x$).
- Integrals inversion (inner integral's singularity).
- Decomposition of the integration domain.
- Duffy's coordinate transform.
- Regular integrand (usual integration techniques).

Sauter S. and, Schwab C., *Boundary Element Methods*, 2011.

Cases to distinguish

- Identical panels.
- Common edge.
- Common vertex.
- When the triangles do not intersect.

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Identical panels

$$k_1(x,y) = w_i(x)G(x,y)w_j(y)$$
$$k_2(\hat{x},\hat{y}) = k_1(\chi_\tau(\hat{x}),\chi_\tau(\hat{y}))g_\tau(\hat{x})g_\tau(\hat{y})$$

$$I = \int_{[0,1]^4} a^3 b^2 c \Big[\, k_2^+(a, a\cdot(1-b+b\cdot c), a\cdot b\cdot c\cdot d, a\cdot b\cdot c) + k_2^+(a, a\cdot b(1-c+c\cdot d), a\cdot b\cdot c, a\cdot b\cdot c\cdot d) + k_2^+(a(1-b\cdot c\cdot d), a\cdot(b-b\cdot c\cdot d), -a\cdot b\cdot c\cdot d, a\cdot b\cdot c\cdot(1-d))\Big]$$

Features available

- Piecewise constant basis functions.
- 3-D Laplace and Helmholtz kernels.
- It works ! (Eric talk)

Perspectives

- Piecewise linear basis functions and higher orders.
- Double layer potential support.
- Explicit formulae (Lenoir-Salles).
- Code optimisation.